EVALUATION OF THE DAMPING CAPACITY OF A COLD-ROLLED SAE 1020 STEEL

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EVALUATION OF THE DAMPING CAPACITY

OF A COLD-ROLLED SAE 1020 STEEL

by

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PREFACE

Since the first observation of the ability of a material to absorb vibrational energy, engineers and physicists have looked upon this property as the possible key to the behavior of materials under dynamic loading conditions. A great amount of work, particularly in the last half century, has been done with the purpose in mind of establishing a quantitative correlation between these two phenomena. The mere evaluation of the specific energy absorption property has proven no small task. The methods used and units employed in describing this property are many and varied so that the uninitiated reader tends to become confused.

It is the aim of this paper to demonstrate a method of calculating the specific energy absorption capacity of a given material. This method incorporates the methods of several other experimenters; however, it differs in the mode of support of the vibrating member.

The writer wishes to thank Professors E. K. Gatcombe, R. E. Newton, and A. K. Schleicher for their technical assistance and cooperation and Machinist Reuel P. Kennicott for his valuable assistance in the fabrication and setup of the experimental equipment.

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TABLE OF SYMBOLS AND ABBREVIATIONS

A	- Amplitude measured on decay trace. Arbitrary units.
E	- Young's Modulus in tension. PSI.
I_z	- Plane moment of inertia. $IN.^4$
K	- Constant of proportionality.
M	- Bending moment. Inch-pounds.
N	- Damping exponent.
U	- Total strain energy. Inch-pounds.
b	- Base dimension of a rectangular section. Inches.
h	- Height dimension of a rectangular section. Inches.
l	- Length. Inches.
n	- Number of cycles.
α	- Logarithmic decrement of strain.
α_U	- Logarithmic decrement of strain energy.
ΔU	- Change in total strain energy per cycle. Inch-pounds per cycle.
Δu	- Change in specific strain energy per cycle (Specific damping capacity) Inch-pounds per cubic inch per cycle.
ϵ	- Strain. Inches per inch.
σ	- Stress. PSI.

CHAPTER I

INTRODUCTION

1. Background.

The damping capacity of a material is a measure of the ability of that material to absorb vibrational energy so that when vibrations are induced in a member, the amplitude of vibrations decreases with time without further energy supply, until they become effectively zero.

For the purpose of this paper, it will suffice to say that the energy is dissipated internally within the material by internal friction caused by displacements of the crystalline structure under load. As yet, the true nature of internal friction is a point of great conjecture amongst engineers and physicists alike. Many theories have been advanced but no firm conclusions as to the microscopic behavior of materials in dissipating this vibrational energy have been reached. This paper is not intended to attempt an explanation of the phenomenon but merely to evaluate its macroscopic effect as it concerns the engineer.

The damping capacity of a material makes itself known in many ways --- the visual observation of decrease in amplitude of a vibrating spring, the phenomenon of "shaft whirl" where damping causes a lateral deflection of a rotating shaft, the increase in temperature of a metal member when subjected to cyclic loading of constant amplitude, the limited amplitude of vibration of a member when excited by a periodic force at its resonant or natural frequency. It is by means of these outward manifestations of the damping properties of materials that damping measurements are made.

In general there are six methods that have been used in the past to measure damping capacity. These are:

1. Decay of free torsional vibrations.
2. Measurement of energy input required to maintain vibrations at constant amplitude.
3. Measurement of heat generated in a specimen vibrated at constant amplitude.
4. Lateral deflections of rotating shafts.
5. Forced vibrations at or near the resonant or natural frequency of the specimen.
6. Hysteresis loop in a static stress-strain curve.

Of these methods numbers 1 and 5 have been used to a greater extent due to the ease of measurement of the required information. The first recorded measurement of damping capacity made in 1906 used the method of decay of free torsional vibrations. In recent years Foppl (4), Lazan (5), Brophy (1), and Cochardt (2) have used this method with good agreement in results. This method is especially good if a thin-walled cylinder is used since the stress distribution is then very nearly constant.

The forced vibrations technique has been used principally by Lazan.

The units of measurement of damping capacity are quite numerous, each experimenter seeming to prefer a different name depending upon his method of measurement. This makes reading on the subject rather difficult since the reader does not know exactly what the writer is referring to when he speaks of the damping capacity of a specific material. Some of the more common units are:

1. Viscosity.
2. Damping capacity.
3. Constant of internal friction.
4. Hysteretic constant.
5. Specific damping capacity.
6. Logarithmic decrement.
7. Elastic phase constant.
8. Coefficient of internal friction.

In order to remove ambiguity from this paper any references to damping capacity will be interpreted as "specific damping capacity" measured in inch-pounds per cubic inch of material per cycle of vibration.

2. Experimental Considerations.

In any vibrating system there are several ways in which energy may be dissipated to produce damping in the system other than the internal dissipative ability of the material under investigation. The most evident ones are aerodynamic damping due to motion of the system in a fluid medium, an energy loss due to energy transfer to the fluid medium by means of acoustical radiation, and most important damping will occur in the support due to elastic deformation of the support and/or relative motion of the vibrating member with respect to the support. Therefore, any attempt at measuring damping capacity must assure that these exterior energy losses are of negligible magnitude or that their quantitative effect can be taken into account.

The method used for the determination of any physical property of a material must assure that the material only is being tested and not the particular geometric configuration the experimenter may be using. This

requirement becomes of significance when we consider that the energy content or strain energy of a loaded member is a function of the stress level. If we are to measure the change in energy we must know how the stress is distributed within the loaded member. The method of loading and mode of support have a great effect on this stress distribution and therefore must be taken into account.

3. Choice of Experimental Method.

The method of measurement decided upon by the writer was to determine the rate of decay of free transverse vibrations of a cantilever beam. In order to minimize the support losses the specimens were made in the form of tuning forks. The symmetry of this type of specimen makes it possible to consider support losses negligible since deflections at the support are extremely small.

While decreasing the support losses, the tuning fork specimen tends to increase the aerodynamic losses due to the relatively broad flat surfaces normal to the direction of motion. The tendency of a tuning fork to produce an audible tone when set into vibration also indicates that acoustical radiation losses could be high. Since both these losses are dependent upon the density of the medium in which the specimen is vibrating, the experiment was carried out in an evacuated chamber in order to minimize these losses.

The cantilever beam, a relatively simple mode of support for point by point stress analysis, unfortunately has a rather complex stress distribution. This particular variation is accounted for in the mathematical approach to the problem.

The material tested in this work was a cold-rolled SAE 1020 steel.

4. Summary of Results.

The only information available for comparison on the damping capacity of a 1020 steel was a curve obtained by Lazan. There is no indication, however, of the condition of this specimen at the time of test. Heat treatment and working of the material produce changes in the damping capacity as well as the other physical properties so it is impossible to make a quantitative comparison between the two curves. The value of the damping exponent (slope of the damping capacity versus stress level curve) obtained by the writer does, however, fall within the range of values given for similar steels.

It was determined for the amplitude of vibrations used in this work that the losses due to aerodynamic damping and acoustical radiation were negligible at atmospheric pressure.

CHAPTER II

EXPERIMENTAL SETUP AND PROCEDURE

1. General.

The requirement of working in an evacuated chamber to decrease aerodynamic damping and acoustical radiation losses necessitates a more complicated setup than would normally be required.

For the evacuated chamber a 17-inch diameter bell jar was used which rested on a steel base plate. The base from a hermetically sealed transformer was set into the base plate to permit bringing in electrical leads without destroying the vacuum seal. On the base plate were mounted a specimen support, a solenoid for actuating the triggering linkage, and the triggering linkage itself.

2. Specimen Support.

The specimen support consisted of a steel block two inches square and four inches high. It was adjustable in the vertical direction for a total travel of one half inch. The top of the block was machined to receive the handle of the tuning fork specimen and a top plate, similarly machined, was then bolted down to hold the tuning fork securely in place.

3. Triggering Solenoid.

The triggering solenoid was a 450 volt A.C. solenoid with a sliding core energized from the lighting circuit through a step-up transformer. When the solenoid was energized the triggering linkage was actuated to rapidly withdraw a spreader from between the tines of the tuning fork specimen.

3. Triggering Linkage.

The triggering linkage consisted of a single bar pivoted at the lower end and attached above the pivot point to the sliding solenoid core. The spreader was rigidly attached to the upper end of the bar. (See Figures 1 and 2.)

4. Specimen.

The specimen was an accurately machined tuning fork. Machining tolerances were such as to give a frequency difference between tines of less than .1%. The dimensions of the tines were as follows:

Length	4.000"
Width	.500"
Thickness	.125"

A lip was machined on the inner surfaces of the tines to permit smoother disengagement of the spreader. (See Figure 3.)

5. Measured Physical Quantity.

For determining the rate of decay of vibrations, there are two physical quantities which can be measured directly. These are the deflection of the vibrating member or the surface strains produced by the deflection. The decision was made to measure the surface strains since equipment was available for detecting them to a high degree of accuracy and for transforming them to useable electrical signals.

The detection of strain was accomplished by mounting a Baldwin SR-4 type A-1 strain gage on the tuning fork tine. This gage was mounted 3.500" from the free end of the tine in order to give a maximum signal. (Note: Physical dimensions of the gage precluded mounting at the point

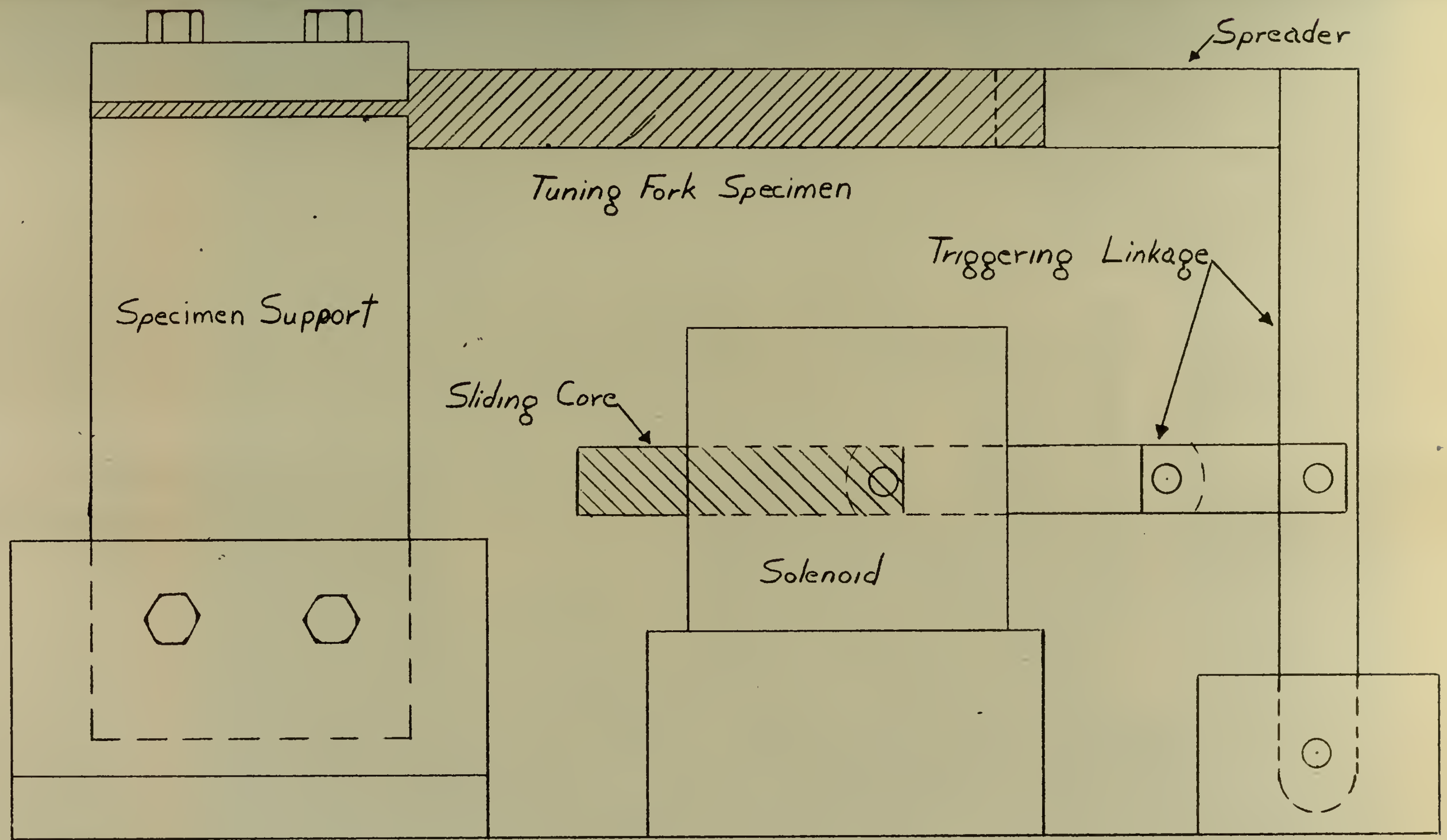


FIGURE (1)

Sketch of Specimen Support and Triggering Linkage

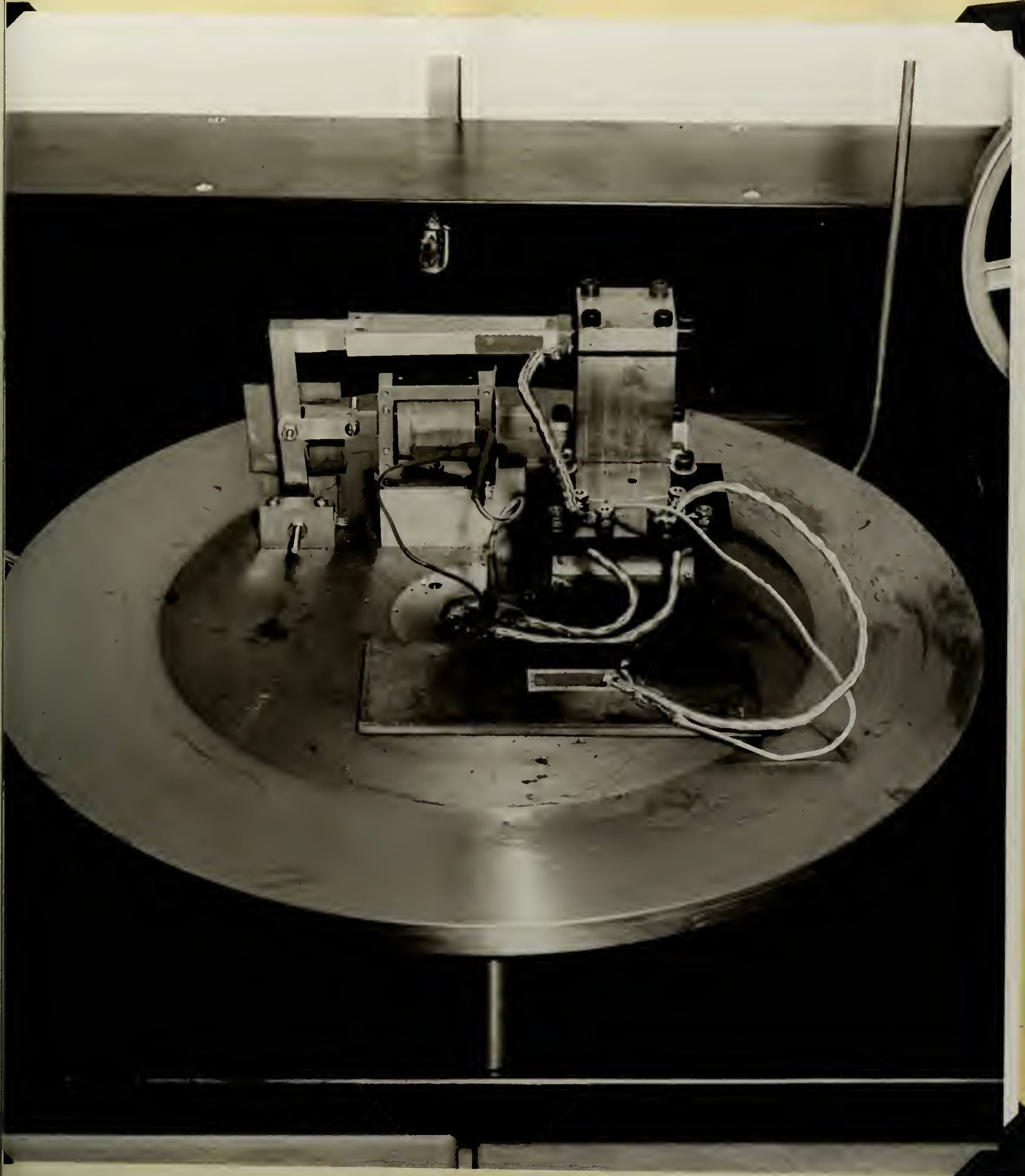


FIGURE (2)
Photograph showing specimen support and triggering linkage

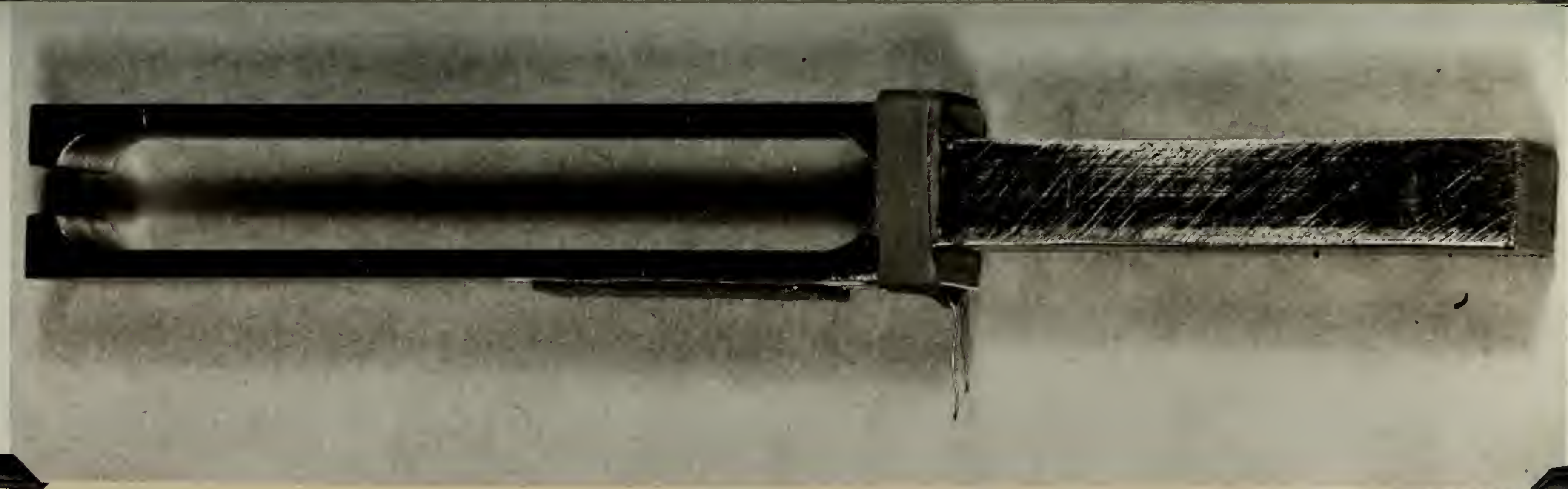


FIGURE (3)
Photograph of tuning fork showing lips on inner tine faces.

FIGURE (3a)
Photograph of tuning fork showing strain gage location.

of maximum strain in the tine.) A gage for temperature compensation was mounted on a similar unstressed piece of material.

6. Instrumentation.

Accurate instrumentation of any experimental setup is usually one of the biggest problems faced by the experimenter. The case at hand was no exception. The first attempts were to use a cathode ray oscilloscope to present a visual picture of the required information. Since the signals from the strain gage are of extremely low voltage, it was necessary to use a high gain amplifier between the gage and the oscilloscope. The difficulty encountered here is that the high amplification required also produces considerable noise which distorts the scope trace. Several different amplifiers and filtering networks were tried before this method was finally abandoned.

The method finally adopted was to use the Hathaway S-15B oscillograph. The Hathaway is a combined electronic, optical, photographic system wherein the strain gage signals are electronically amplified and then fed into a very sensitive ballistic galvanometer. The movable element of the galvanometer carries a mirror which deflects a beam of light as the movable element is deflected in correspondence with the strain gage signals. This light beam is focused on a moving sheet of photographic paper which is then developed to produce a permanent record. (See Figure 4 for a sample record obtained in this manner.)

The photograph in Figure 5 shows the complete setup used for this paper.

7. Procedure.

The procedure followed for obtaining a record of the decay curve was as

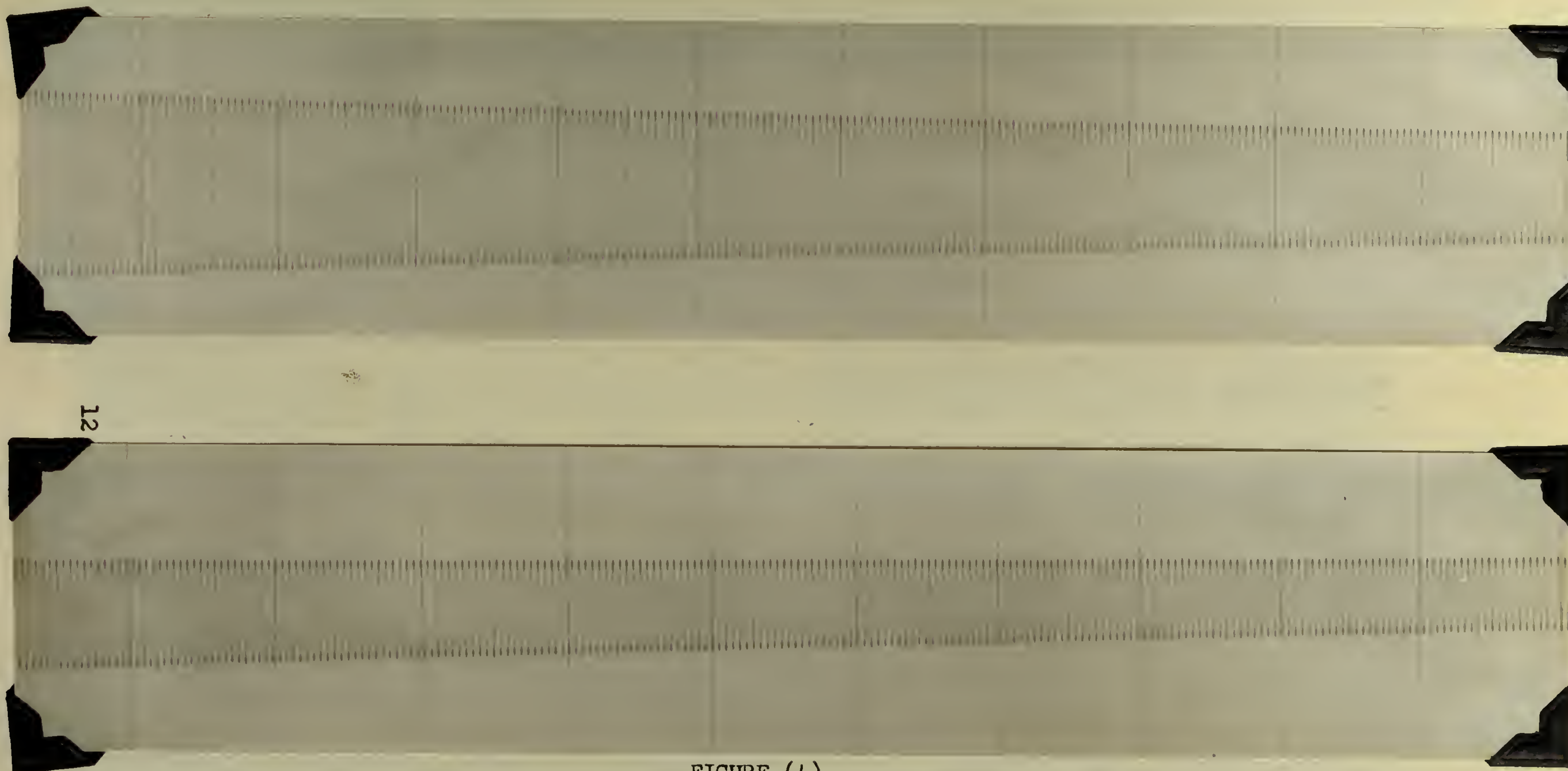


FIGURE (4)
Sample decay trace obtained with Hathaway Oscillograph.

FIGURE (5)
Photograph of complete test setup.

follows:

1. Perform all balancing and calibration of detecting and recording equipment.

2. Insert spreader between tines of specimen and evacuate bell jar.

3. With oscillograph photographic paper traveling at maximum speed, energize solenoid to remove spreader from tines. Maximum speed is used to permit maximum discrimination between cycles.

4. Develop photographic paper.

8. Analysis of Decay Curve.

The decay curve was analyzed to determine the logarithmic decrement. The logarithmic decrement is defined as the logarithm of the ratio of two successive amplitudes. Due to the possibility of a very large reading error occurring in measuring the amplitude of two successive cycles of very nearly the same height, an approximating formula was used where the decrement was averaged over a number of cycles. The approximating formula used in this instance was $\alpha = \frac{2}{n}\left(\frac{A_m - A_{m+n}}{A_m + A_{m+n}}\right)$ where n is the number of cycles between observations and A_m and A_{m+n} are the observed amplitudes. The error introduced by this approximation is less than 1% provided the ratio of amplitudes is less than 1.4.

The amplitude measurements were made with an optical comparator. The reading accuracy of this instrument was .002 inches.

Logarithmic decrements were calculated over the range of stress from approximately 17,000 psi to 3,000 psi to determine the variation of decrement with stress level. This range of stress was available from a single decay trace due to the decrease in stress level as the vibrations decay.

CHAPTER III

CALCULATION OF SPECIFIC DAMPING CAPACITY

Basically, the method used in this work for calculation of specific damping capacity was to determine the specific energy absorption per cycle of vibration.

The information obtained from the decay of free vibrations makes possible the calculation of the logarithmic decrement of strain (α) defined as:

$$\alpha = \ln \frac{A_m}{A_{m+1}}$$

Since we are interested in an energy change, it is necessary to convert this to an energy decrement.

The logarithmic decrement of strain energy is defined by:

$$\alpha_U = \ln \frac{U_m}{U_{m+1}}$$

Since strain energy is directly proportional to the square of strain, the decrement of strain energy becomes:

$$\alpha_U = \ln \frac{U_m}{U_{m+1}} = \ln \left(\frac{A_m}{A_{m+1}}\right)^2$$

or $\alpha_U = 2 \ln \frac{A_m}{A_{m+1}} = 2\alpha$

With this relation, the energy absorption per cycle becomes:

$$\Delta U = U_m - U_{m+1} = U_m\left(1 - e^{-2\alpha}\right)$$

where U_m is the strain energy for the mth cycle of vibration.

The calculation of U_m is accomplished by means of the theorem of Castigliano:

$$U_m = \int_0^l \frac{M_{(x)}^2}{2EI_z}\,dx$$

The bending moment M(x) for a freely vibrating beam is a rather complex expression since it is a bending moment caused by the inertia forces. However, the assumption of a linear variation of bending moment such as would be obtained in a cantilever loaded at the free end by a concentra-

ted load, will not produce an appreciable error. With this assumption, evaluation of Castigliano's theorem becomes:

$$U_m = \frac{E\epsilon_{max}^2 I_z \ell}{1.5h^2}$$

The expression of the change in strain energy now becomes:

$$\Delta U = \frac{E\epsilon_{max}^2 I_z \ell}{1.5h^2}\left(1-e^{-2\alpha}\right)$$

or, converting to units of stress:

$$\Delta U = \frac{I_z \ell}{1.5h^2 E}\left(1-e^{-2\alpha}\right)\sigma_{max}^2 \qquad (1)$$

Equation (1) now enables us to calculate the total change in strain energy of the cantilever. It includes the effects of type of loading and support and, therefore, does not relate to a specific property of the material.

If we now consider an infinitely small volume of material dV, we may assume that the energy absorbed in this volume to be

$$d\Delta U = \Delta u \, dV \qquad (2)$$

where Δu is a measure of the specific energy absorption ability of the material (specific damping capacity) per unit volume per cycle. Integrating, this becomes:

$$\Delta U = \int \Delta u \, dV \qquad (3)$$

B. J. Lazan (5) in his extensive work on the damping capacity of engineering materials has produced consistent results which indicate that the specific damping capacity is related to the stress in the following manner:

$$\Delta u = K\sigma^N \qquad (4)$$

where K and N are constants depending on the particular material. Assuming this equation to be valid, upon substitution in equation (3) we obtain:

$$\Delta U = K\int \sigma^N dV \qquad (5)$$

In order to evaluate this integral, we employ the bending stress equation from elementary strength of materials:

$$\sigma = \frac{M(x)\,y}{I_z} \qquad (6)$$

making the same assumption of linear variation of bending moment as was previously made.

$$M(x) = M_{max}\left(\frac{x}{l}\right) \qquad (7)$$

This final substitution into equation (5) gives the following integral which can be integrated:

$$\Delta U = K\int \frac{M_{max}^N\, x^N y^N}{l^N I_z^N}\, dV \qquad \text{or}$$

$$\Delta U = \frac{2KbM_{max}^N}{l^N I_z^N}\int_0^l \int_0^{\frac{h}{2}} x^N y^N\, dx\, dy$$

Performing the integration and making indicated substitutions (Appendix 1), we arrive at the following expression which relates the specific damping capacity of the material to the total energy absorption per cycle in the vibrating member:

$$\Delta U = \frac{\Delta u V}{(N+1)^2} \qquad (8)$$

Solving for Δu:

$$\Delta u = \frac{(N+1)^2 \Delta U}{V} \qquad (9)$$

ΔU can be calculated from equation (1); however, equation (9) for converting to specific damping capacity cannot be solved since it contains the factor $(N+1)^2$ where N is still an unknown.

This difficulty can be overcome by referring back to Lazan's equation. This equation, when plotted to logarithmic coordinates would produce a straight line of slope N. If the derived equation (9) is equivalent to Lazan's equation the same result should be obtained from it.

Investigation of equation (9) reveals that due to the manner of occurrence of the unknown factor N therein, this factor would have no effect on the slope of a specific damping capacity versus stress curve plotted from it. It is this fact which enables us to determine the damping exponent N, by a simple graphical solution.

The graphical solution proceeds as follows: An assumed value of $N = N'$ is substituted into equation (9) and a fictitious value of $\Delta u = \Delta u'$ obtained. $\Delta u'$ is then plotted against stress to logarithmic coordinates and the slope of the resultant straight line measured. The slope of this line is then the true value N. Next the $\Delta u'$ intercept is determined and corrected by the factor $\left(\frac{N+1}{N'+1}\right)^2$ to determine the true Δu intercept. Then the line obtained from the plot of $\Delta u'$ versus stress is transposed parallel to itself until it intersects the Δu axis at the corrected value of $\Delta u'$. The resultant line is now the correct plot of specific damping capacity versus stress, and the Δu intercept is the constant K in Lazan's equation. This method of solution is illustrated in figure (6) with $N'=2$.

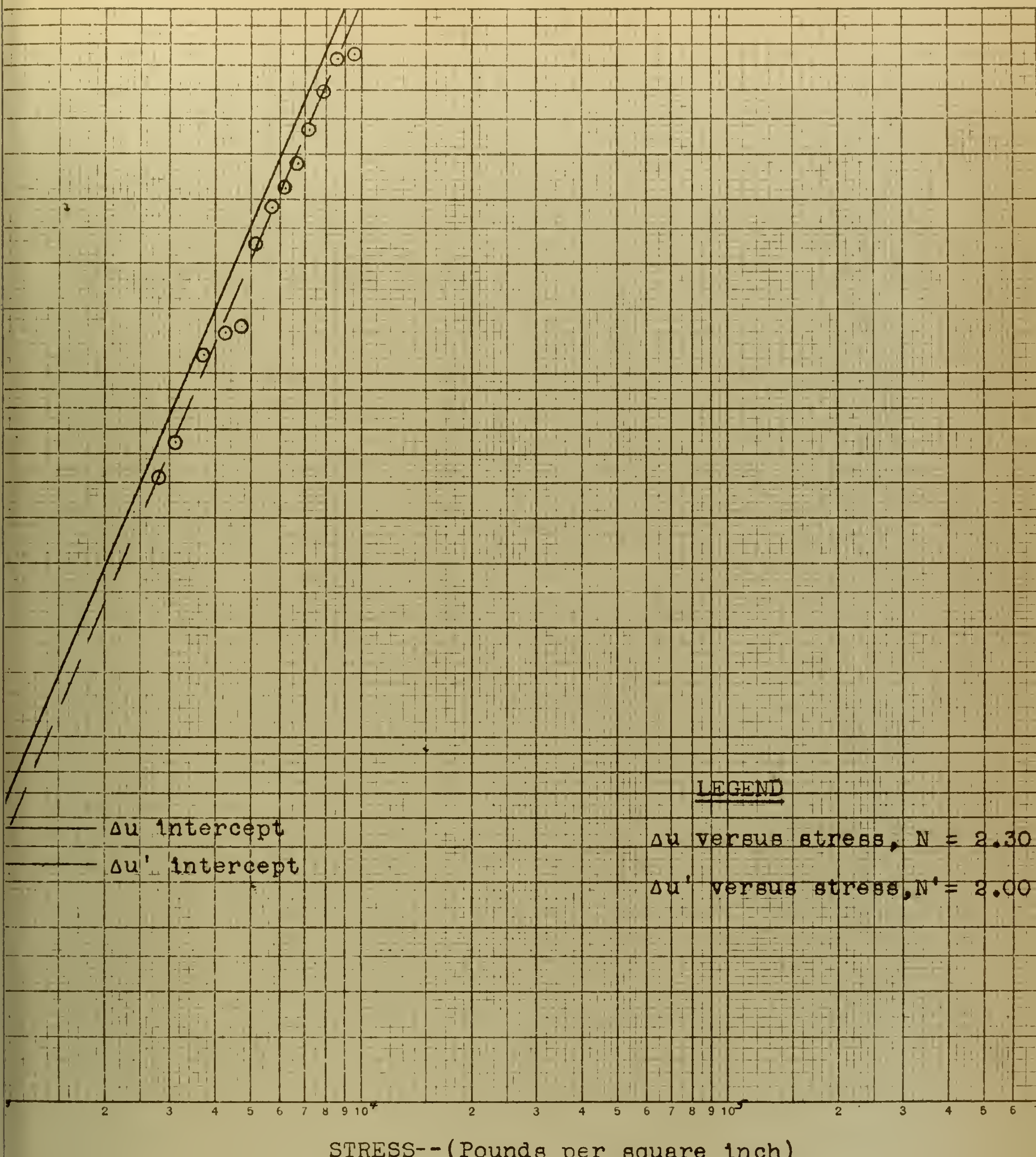


FIGURE (6)
Illustration of the graphical method for determination of the damping exponent (N).

CHAPTER IV

PRESENTATION OF RESULTS AND CONCLUSIONS

1. Results.

The results of this paper, i.e., the damping capacity of a cold-rolled SAE 1020 steel, are presented as the curves of Figures (8), (9), and (10). Runs #1 and #2 were made in an evacuated chamber in order to minimize the losses due aerodynamic damping and acoustical radiation. Run #3 was madé at atmospheric pressure for purposes of comparison. The equations relating specific damping capacity to stress derived from the three curves are:

$$\#1.\ \Delta u = 5.25 \times 10^{-5}\ \sigma^{2.30}$$

$$\#2.\ \Delta u = 5.28 \times 10^{-5}\ \sigma^{2.26}$$

$$\#3.\ \Delta u = 5.40 \times 10^{-5}\ \sigma^{2.29}$$

These three equations agree within the limits of experimental error. The difference between the runs made at low pressure and the runs at atmospheric pressure is not large enough to be of significance other than the normal scatter and reading error.

The exponential relationship of Lazan's equation is demonstrated quite definitely in all three cases.

The point by point evaluation of damping capacity at the various stress levels is consistent for the three runs.

The lack of published data on the damping capacity of specific materials make any quantitative comparison of the results of this paper impossible. Lazan has published curves of the damping capacity of several steels, one of them being a 1020 steel; however, the condition of the metal at the time

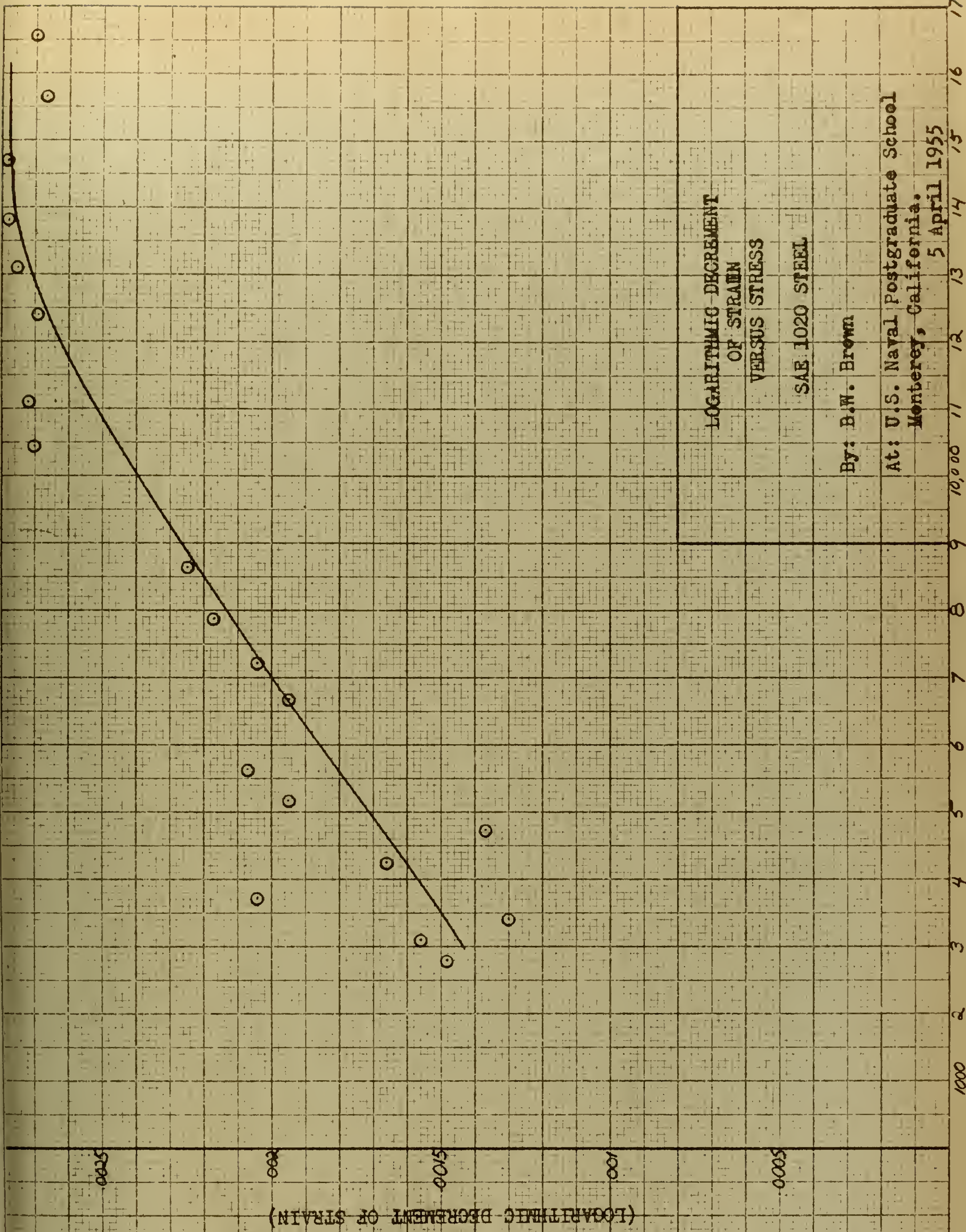
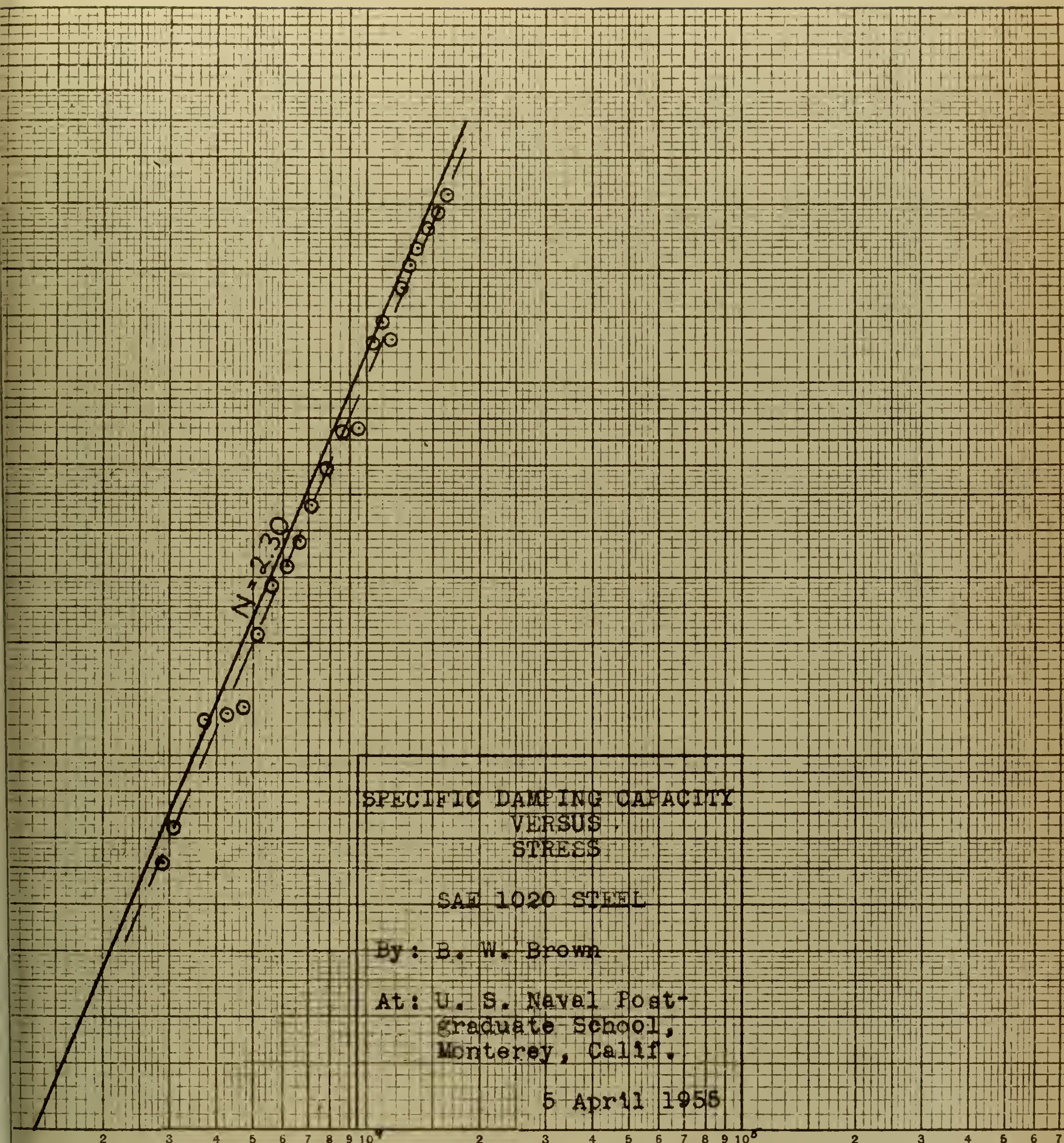


FIGURE (7)

Variation of decrement of strain with stress.

RUN ONE



STRESS-- (Pounds per square inch)

FIGURE (8)

Variation of specific damping capacity with stress at a pressure of .02 inches of Hg.

RUN TWO

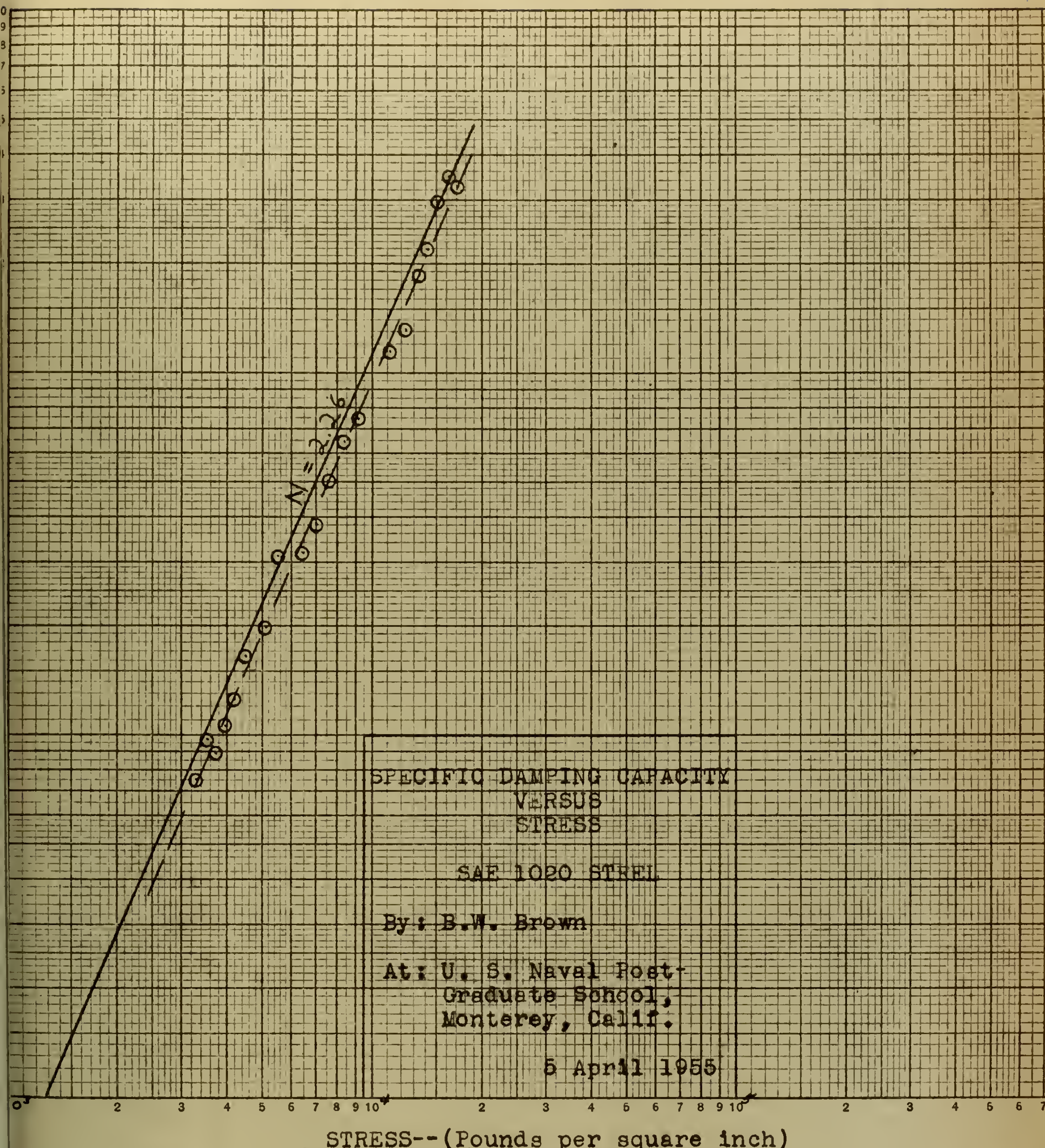
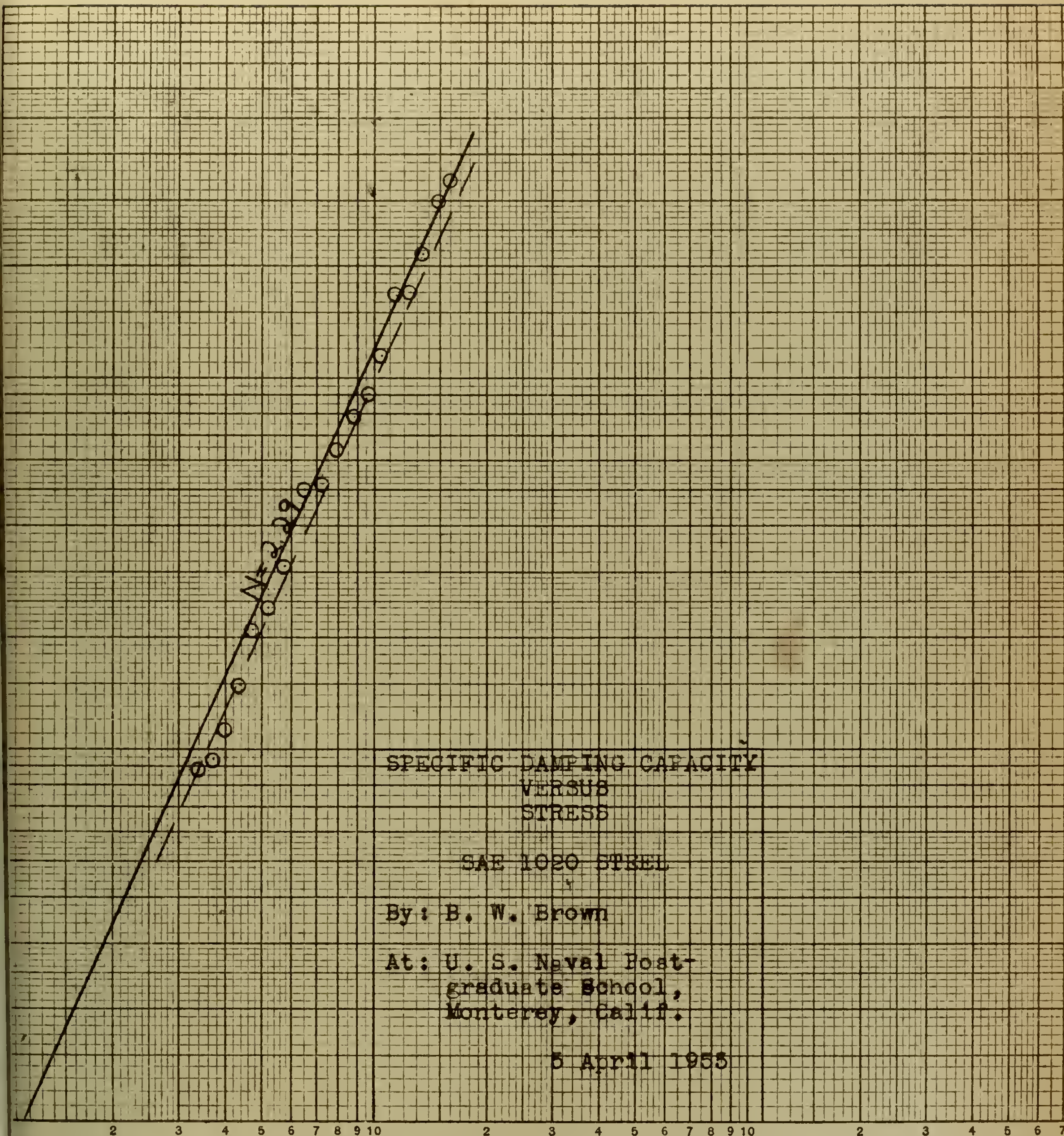


FIGURE (9)
Variation of specific damping capacity with stress at a pressure of .03 inches of Hg.

RUN THREE



STRESS--(Pounds per square inch)

FIGURE (10)
Variation of specific damping capacity with stress at atmospheric pressure.

of test as to heat treatment or forming method is not given. Since we would expect changes in damping capacity with heat treatment and forming just as in the other physical properties, Lazan's data is not complete enough for comparison purposes.

2. Conclusions.

The conclusions reached on the basis of these results are:

1. Lazan's equation relating damping capacity to stress level is valid.
2. The employment of a tuning fork as the vibrating member for determining damping capacities will produce consistent results.
3. For the amplitude of vibrations used in this work, the losses due to aerodynamic damping and acoustical radiation are negligible.

3. Sources of Error.

The most probable source of error in this experimental setup is in the analysis of the decay trace to determine the logarithmic decrement. The limited speed of the photographic paper in the Hathaway oscillograph does not permit adequate sharp definition of the peak of each cycle for accurate measurements. The use of a tuning fork designed for a much lower natural frequency would remove a large proportion of this source of error.

BIBLIOGRAPHY

1. Brophy, G. R. — DAMPING CAPACITY - ITS VARIATION AND RELATION TO OTHER PROPERTIES Transactions of American Society of Metals, Vol. 24, 1936, p. 917.

2. Cochardt, A. W. — A METHOD FOR DETERMINING THE INTERNAL DAMPING OF MACHINE MEMBERS Journal of Applied Mechanics, Trans. ASME, Vol. 21, 1934, p. 257.

3. Cochardt, A. W. — ORIGIN OF DAMPING IN HIGH-STRENGTH FERROMAGNETIC ALLOYS Journal of Applied Mechanics, Vol. 20, 1953, p.196.

4. Foppl, O. — THE PRACTICAL IMPORTANCE OF THE DAMPING CAPACITY OF METALS, ESPECIALLY STEELS, Journal Iron and Steel Inst., Vol. 134, 1936, p. 393.

5. Lazan, B. J. — DAMPING, FATIGUE AND DYNAMIC STRESS-STRAIN PROPERTIES OF MILD STEEL Journal of Applied Mechanics, Trans. ASME, Vol. 51, 1951, p. 649.

6. Potter, E. V. — DAMPING CAPACITY OF METALS Bureau of Mines R.I. 4194, March 1948

7. Timoshenko, S. P. — STRENGTH OF MATERIALS, PART II D. Van Nostrand, 1942

8. Zener, C. — ELASTICITY AND ANELASTICITY OF METALS The University of Chicago Press, 1948

APPENDIX I

EVALUATION OF SPECIFIC DAMPING CAPACITY INTEGRAL

In Chapter III the expression

$$\Delta U = \frac{2 K b M_{max}^N}{\ell^N I_z^N} \int_0^{\ell} \int_0^{\frac{h}{2}} x^N y^N dx\, dy$$

was developed. Integrating this expression and applying the limits we obtain:

$$\Delta U = \frac{2 K b M_{max}^N}{\ell^N I_z^N} \left[\frac{\ell^{N+1}}{N+1} \times \frac{h^{N+1}}{2^{N+1}(N+1)} \right]$$

Performing the indicated cancellations this expression becomes:

$$\Delta U = \frac{K b M_{max}^N}{I_z^N} \left[\frac{\ell h^{N+1}}{2^N (N+1)^2} \right]$$

If this expression is now factored in the following manner:

$$\Delta U = K (b \ell h) \left(\frac{M_{max} h}{2 I_z} \right)^N \frac{1}{(N+1)^2}$$

it is seen that the term (blh) represents the volume of the rectangular cantilever, and the term $K \left(\frac{M_{max} h}{2 I_z} \right)^N$ where $\left(\frac{M_{max} h}{2 I_z} \right)^N$ is σ_{max}^N

represents Lazan's equation for specific damping capacity:

$$\Delta u = K \sigma^N$$

Performing these substitutions, we obtain equation (8)

$$\Delta U = \frac{\Delta u \, V}{(N+1)^2} \qquad (8)$$

APPENDIX II

Experimental Data and Sample

Calculations

RUN #1

Pressure = .02" hg

A_m	A_{m+n}	n	α	σ	$\Delta u'$
39.80	37.50	22	.00270	16,550	.0320
37.50	35.45	21	.00267	15,620	.0284
35.45	33.35	22	.00278	14,700	.0260
33.35	31.45	21	.00278	13,800	.0229
31.45	29.60	22	.00275	13,100	.0206
29.60	27.90	22	.00269	12,400	.0179
27.90	26.65	21	.00218	11,750	.0130
26.65	25.10	22	.00272	11,100	.0145
25.10	23.65	22	.00270	10,450	.0128
23.65	21.70	43	.00190	9,550	.0075
21.70	19.65	44	.00225	8,650	.0073
19.65	17.90	43	.00217	7,880	.0059
17.90	16.40	43	.00204	7,220	.00464
16.40	15.15	44	.00195	6,670	.00375
15.15	14.05	40	.00188	6,180	.00311
14.05	12.80	45	.00207	5,630	.00286
12.80	11.75	44	.00195	5,170	.00226
11.75	10.75	65	.00137	4,730	.00134
10.75	9.65	65	.00166	4,250	.00129
9.65	8.45	65	.00204	3,720	.00123
8.45	7.80	65	.00130	3,410	.00100
7.80	7.05	65	.00156	3,100	.000645
7.05	6.40	65	.00149	2,820	.000518

RUN #2

Pressure = .03" hg

A_m	A_{m+n}	n	α	σ	$\Delta u'$
41.95	39.65	22	.00256	17,100	.0323
39.65	37.10	21	.00316	16,150	.0343
37.10	34.85	21	.00298	15,100	.0299
34.85	33.05	21	.00252	14,200	.0219
33.05	31.50	20	.00236	13,450	.0185
31.50	30.25	20	.00202	12,300	.0131
30.25	27.50	41	.00232	11,200	.0124
27.50	24.45	42	.00284	9,960	-
24.45	22.40	42	.00208	9,120	.0074
22.40	20.40	43	.00217	8,320	.0064
20.40	18.65	44	.00201	7,600	.0050
18.65	17.30	43	.00175	7,050	.0038
17.30	15.80	44	.00206	6,430	.00318
15.80	14.85	43	-	-	-
14.85	13.55	44	.00208	5,520	.00311
13.55	12.55	43	.00178	5,120	.00199
12.55	12.00	44	-	-	-
12.00	11.05	44	.00187	4,500	.00163
11.05	10.30	43	.00164	4,200	.00126
10.30	9.65	44	.00159	3,930	.00107
9.65	9.05	43	.00149	3,680	.00088
9.05	8.65	44	.00179	3,520	.00097
8.65	8.05	44	.00163	3,280	.00075

RUN #3

Pressure = 30.04" hg

A_m	A_{m+n}	n	α	σ	$\Delta u'$
39.60	37.12	22	.00290	16,000	.0340
37.12	33.20	25	.00301	14,900	.0298
33.20	31.60	22	.00285	13,600	.0215
31.60	30.30	21	.00221	12,500	.0170
30.30	28.20	20	.00253	11,400	.0168
28.20	27.30	42	.00215	10,500	.0115
27.30	24.50	43	.00205	9,700	.0090
24.50	21.00	43	.00201	8,850	.0078
21.00	19.12	44	.00190	8,000	.0064
19.12	17.31	44	.00181	7,250	.0052
17.31	15.80	43	.00204	6,500	.0050
15.80	13.75	44	.00176	5,740	.0031
13.75	12.50	43	.00175	5,200	.0024
12.50	11.18	43	.00160	4,700	.0021
11.18	10.40	44	.00172	4,350	.00150
10.40	9.72	43	.00152	4,000	.00113
9.72	8.72	44	.00189	3,700	.00094
8.72	7.87	44	.00141	3,380	.0088

RUN #1

2α	$e^{-2\alpha}$	$1-e^{-2\alpha}$	$\sigma^2 \times 10^{-6}$	$\Delta u'$
.00540	.99461	.00539	273	.0320
.00534	.99465	.00535	244	.0284
.00556	.99445	.00555	216	.0260
.00556	.99445	.00555	190	.0229
.00550	.99451	.00549	172	.0206
.00538	.99463	.00537	154	.0179
.00436	.99565	.00435	138	.0130
.00544	.99455	.00545	123	.0145
.00540	.99460	.00539	109	.0128
.00380	.99620	.00380	91.1	.0075
.00456	.99551	.00449	74.8	.0073
.00434	.99565	.00435	62.2	.0059
.00408	.99590	.00410	52.1	.00464
.00309	.99610	.00390	44.4	.00376
.00376	.99625	.00375	38.2	.00311
.00414	.99585	.00415	31.7	.00286
.00390	.99610	.00390	26.7	.00226
.00274	.99725	.00275	22.4	.00134
.00332	.99670	.00330	18.0	.00129
.00408	.99590	.00410	13.8	.00123
.00312	.99690	.00310	9.6	.000645
.00296	.99700	.00300	7.95	.000518

Sample Tabular Form for Calculation of Specific Damping Capacity ($\Delta u'$)

Thesis 28454
B8097 Brown
Evaluation of the damping capacity of a cold-rolled SAE 1020 steel.